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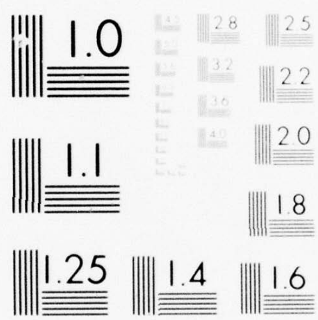

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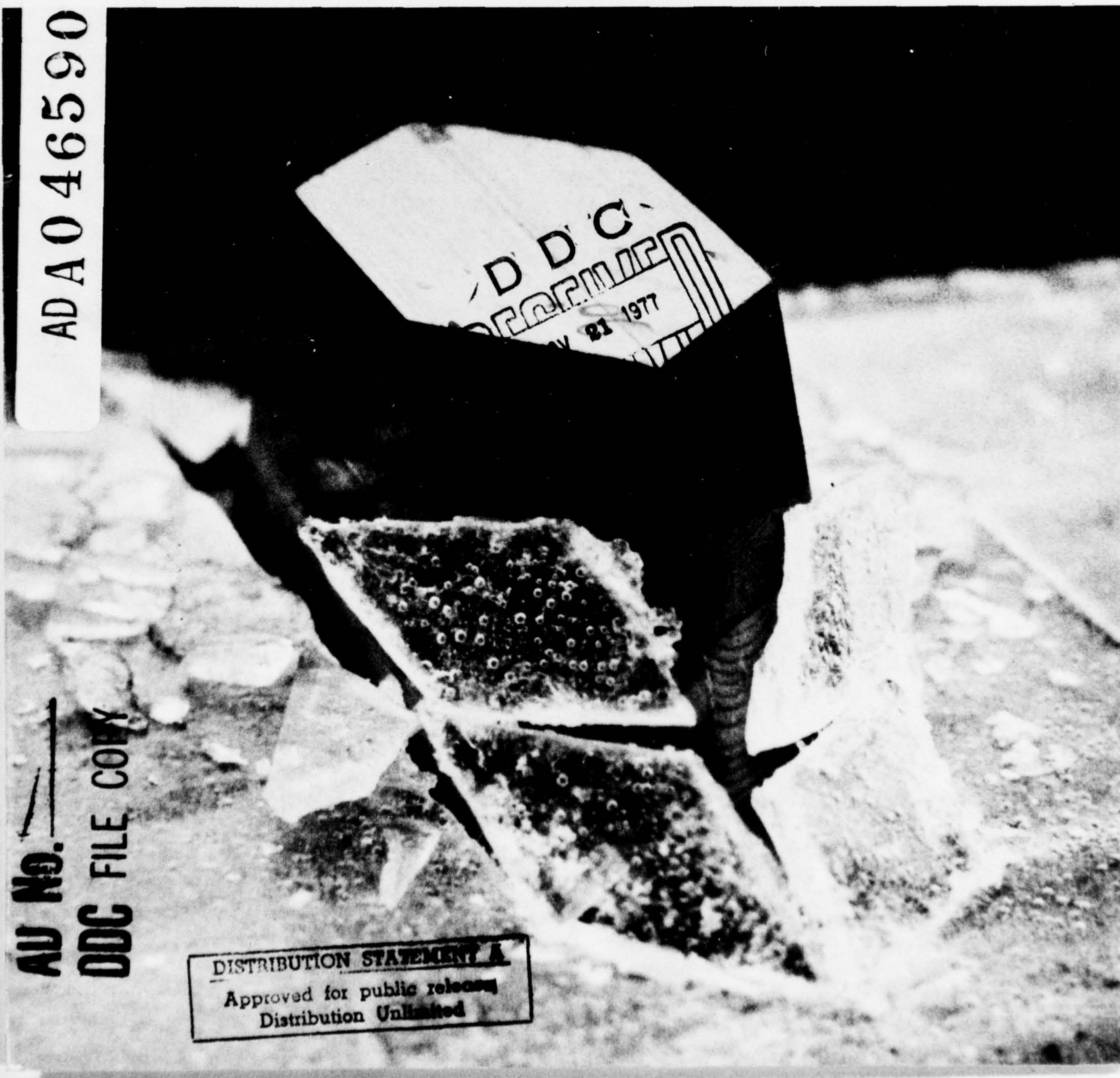

CRREL Report 77-26

Intermittent ice forces acting on inclined wedges

Per Tryde

October 1977

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A theory for ice forces acting on inclined wedges has been developed, thus making it possible to predict the magnitude of the intermittent ice forces from knowledge of the physical parameters of the system. The theory has been verified by model tests with artificial and natural ice.

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PREFACE

This report was prepared by Per Tryde, Associate Professor, of the Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark. It is funded by U.S. Army Corps of Engineers Civil Works Project 31334, *Preventing and Removing Ice from Adhering to Lock Walls and Gates*, and by funds from the Danish Council for Scientific and Industrial Research.

Technical reviewers of this report were Dr. Donald Nevel and Dr. Andrew Assur of CRREL.

The author wishes to thank the staffs of the U.S. Army Cold Regions Research and Engineering Laboratory and of the Institute of Hydrodynamics and Hydraulic Engineering for their very valuable assistance in conducting this investigation. Special appreciation is given to C. Sørensen, now studying for his Ph.D. in ice research, for performing the tests with artificial ice and preparing a report on the results. Also, special thanks are due to Dr. J. Schwarz and Dr. M. Kloppenburg of the Hamburgische Schiffbau-Versuchsanstalt who made some tests with natural ice possible. Some of the calculations and tables have been checked by J. Magtengaard, a student working at ISVA. Finally, the author wishes to extend his thanks to G. Frankenstein of CRREL for the encouragement to continue the work, making it possible to complete the investigation in collaboration with CRREL.

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LIST OF SYMBOLS

Structure:	Width of structure	d (m)
	Inclination of wedge to horizontal	β (degrees)
	Included angle at point of wedge in horizontal plane	2α (degrees)
Ice floe:	Thickness of ice	e (m)
	Velocity of floe	u_c (m/s)
	Velocity of current	U_{water} (m/s)
	Velocity of wind (10 m above surface)	U_{wind} (m/s)
	Distance of line of rupture from face of contact	y (m)
	Eccentricity of horizontal force	x (m)
	Section modulus	W (m^3)
	Area of floe	A (m^2)
Ice properties:	Density of ice	ρ (tonne/m^3)
	Compression strength	r_c (kN/m^2)
	Flexural strength	r_b (kN/m^2)
	Rupture compression strength, combined loading	r_{bc}(kN/m^2)
	Resulting stress	σ_{bc} (kN/m^2)
	Young's modulus	E (kN/m^2)
	Wind shear	τ_a (kn/m^2)
	Water shear	τ_w (kN/m^2)
Forces:	Initial vertical force	P_1 (kN)
	Total horizontal force	F (kN)
	Wind shear	$F_{ex,wind}$ (kN)
	Water shear	$F_{ex,water}$ (kN)
	Vertical force (no friction)	V (kN)
	Vertical force (with friction)	V' (kN)
	Horizontal force on each face (no friction)	T (kN)
	Horizontal force on each face (with friction)	T' (kN)
	$'$ indicates that friction in vertical direction has been included	
Dimensionless coefficients:	Friction	μ
	Reduction factor	C_F
	$C_F = F/(r_c\, e\, d)$	
	Rupture factor	y/e

Dimension factor	e/d
Hydrodynamic mass factor	C_A
Angular movement factor	C_B
Eccentricity factor	f
System factors used in formulae:	
$\frac{C_1}{C_2} \approx \frac{1 - \mu \tan\beta/\sin\alpha}{\mu + \tan\beta/\sin\alpha}$	C_1/C_2
$C_3 = \frac{C_5}{C_2} = 6\frac{e}{d}\cos\alpha + 6\frac{C_1}{C_2}$	C_3
$C = 0.16\sqrt{E/(\rho u_c{}^2 \sin^2\alpha)}$	C
Ratio of bending strength over compression strength $\epsilon = r_b/r_c$	ϵ
Scale ratio	λ

SUMMARY

The maximum force of an ice sheet acting on a structure with vertical faces can be written (for notations see *List of Symbols*) as

$$F_{max} = r_c e d \qquad \text{(kN)}$$

assuming d/e to be so large that the indentation effect is negligible (see *Structures with Vertical Forces*).

The actual intermittent force, a peak load of short duration acting on the inclined wedge, will be reduced to

$$F = C_F F_{max}.$$

C_F is a function of a system of nondimensional data:

$$C_F = \phi\left(\frac{E}{\rho u_c{}^2}, \frac{e}{d}, \frac{y}{e}, \frac{r_b}{r_c}, \alpha, \beta, \mu\right).$$

An approximate formula of C_F, valid in the interval $0 \leq C_F \leq 0.4$, can be written as

$$C_F = \frac{5.2\sqrt[3]{r_b/r_c}}{\sqrt{C}} \qquad \text{(A)}$$

where

$$C = 0.16 \cdot \sqrt{E/(\rho u_c{}^2 \cdot \sin^2\alpha)} \cdot C_1/C_2 \cdot (C_5/C_2)^2 .$$

Equation A is derived by use of a rupture stress analysis and by use of dynamic equations of the system. It is assumed that no other exterior forces are acting on the ice sheet and that $u_c > 0$.

The effect of friction in the vertical direction on the face of the wedge has been included, while friction in the horizontal direction has been neglected, as its influence on rupture is negligible.

In the case of $u_c = 0$, representing a floe resting against the wedge, the formula becomes

$$C_F = \frac{F_{ex}}{r_c e d} .$$

F_{ex} will be either the wind shear or the shear from a current or a combination of the two, represented by the formulae, respectively, where A is the area of the floe (m^2):

$$F_{ex,wind} = 4.8 \cdot 10^{-3} \cdot \frac{1}{2}\,\rho_a \cdot U^2_{wind} \cdot A \qquad \text{(kN)}$$

$$F_{ex,wind} = 5.4 \cdot 10^{-3} \cdot \frac{1}{2}\,\rho_w \cdot U^2_{water} \cdot A \qquad \text{(kN)}$$

where $\rho_a = 1.25 \times 10^{-3}$ tonne/m^3 is the density of air and $\rho_w = 1$ tonne/m^3 is the density of water, which gives a value of $C_F = 1.0$ for $F_{ex} = r_c e d$. For $F_{ex} > r_c ed$ the floe will move continuously and eq A will apply.

The velocity range will be from approximately 0.1 m/s to 4 m/s. The angle α may vary from 30° to 60° and β from 45° to 70°. The friction coefficient will seldom exceed 0.1, but in some cases it may be considerably higher. The value of C_1/C_2 will vary from 0.1 to 1.0 depending on the combination of α, β and μ. Graphs showing the relationships are presented in Figures 7 and 10. The factor e/d can vary from zero to approximately 0.3, limited by the assumptions made for the rupture patterns. The ratio between the bending strength and the compression strength may vary from 0.2 to 0.5.

The influence of ρ, u_c and E is not appreciable, since it is in the fourth root. However, an increase in velocity should give an increase in force. Low values of E should also result in higher values of C_F. An increase in β will result in an increased force. If $\mu \rightarrow 0.3$, C_1/C_2 can reach rather low values, resulting in values of C_F up to 1.0, i.e. no reduction at all. The theoretical curves and the results of the tests are shown in Figure 12.

INTERMITTENT ICE FORCES ACTING ON INCLINED WEDGES

Per Tryde

INTRODUCTION

The research described in this report was initiated at the Institute of Hydrodynamics and Hydraulic Engineering (ISVA),* Technical University of Denmark, by Professor H. Lundgren, as methods for the rational calculation of ice forces acting on structures in the open sea were urgently needed.

In 1971 a simple formula for calculating ice forces was developed for structures with vertical faces which were either perpendicular to the direction of the forces or positioned as wedges with vertical faces. It was soon realized that the design forces could be considerably reduced if inclined faces were introduced, thus resulting in bending failure instead of compression or shear failure. Korzhavin (1971) presented an empirical formula but did not establish the functional relationship of the parameters involved in the system. The aim, therefore, was 1) to present a theory that could explain the phenomenon and 2) to verify the theory by model tests.

The theory was presented in 1973, and has since been improved in accordance with the findings from model tests. Model tests were made in Lyngby with an artificial "ice" material developed by ISVA. Tests with natural ice have been performed by the Hamburgische Schiffbau-Versuchsanstalt (HSVA) in Hamburg, Germany, and by CRREL in Hanover, New Hampshire.

RECENT ICE RESEARCH IN DENMARK

General

This section contains a short review of pertinent ice research performed at ISVA since 1968.

In Denmark, ice forces had hitherto been determined by empirical rules based on the Danish Standards of Loading, indicating a horizontal ice force on a bridge pier corresponding to 1.5 to 3.0 tonnes per meter free span of the bridge in question, the higher value to be used for structures in fiords, sounds, etc.

Recently, construction activity in Danish waters has increased with the building of large bridges interconnecting the main islands, oil piers on the open shores, and offshore lighthouses.

Structures with vertical faces

A general expression for ice force can be written as

$$F = kr_c ed$$

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* *Instituttet for Stromningsmekanik og Vandbygning.*

where k is a dimensionless variable, r_c the compressive strength of the ice, e the ice thickness, and d the width of the structure. For slender vertical structures such as piles and cylinders the value of k can be given as

$$k = \frac{\sigma}{\sigma_\infty} = 1 + 1.5\frac{e}{d} \qquad \text{for } 0 < \frac{e}{d} < 2$$

where σ_∞ is the strength for the case $d \to \infty$. This strength may not be equal to what is normally denoted as the compressive strength, since firm rules for performing "compression" tests are lacking. This problem is at present being considered by the IAHR committee on ice testing. The formula for the force F may therefore have to be corrected accordingly. The effect of ice thickness on compressive strength has been studied by Schwarz (1974), and by Frederking and Gold (1974).

The formula $k = 1 + 1.5\ e/d$ will give values which are too high when e/d is larger than 1, according to investigations by Afanas'ev (1973), Assur (1972) and Schwarz (1974). The following formula will give results between those of Assur (1972) and Afanas'ev (1973):

$$k = 1 + 2.1\left(0.4 + \frac{d}{e}\right)^{-1}$$

Wedges with inclined faces

Korzhavin (1971) has given formulae for ice forces acting on inclined wedges, based on analytical considerations and data obtained from prototype structures. However, a theory has not been presented by Korzhavin. The author attempted to develop such a theory and presented this in a lecture given at the University of Iowa Institute of Hydraulic Research in the summer of 1973, the substance of which is described in Tryde (1973b).

To evaluate this theory, model tests were performed at ISVA. These tests were made in a 2-m wide flume with a floe of approximately 8 m^2, of which approximately 2.5 m^2 consisted of an artificial material composed as stated below and the remaining part consisted of plywood. The model floe was run against a wedge-faced object on a flowing current. In these tests the Froude number was approximately 0.2, with a floe velocity of 0.3 m/s, an ice thickness of approximately 1 cm, and a wedge width of 16 cm. The object had an included angle at the point of the wedge of 100° in the horizontal plane and an inclination of 55° to horizontal. The wedge could be rotated ± 10° in order to vary the angle of inclination. The wedge was mounted on a dynamometer equipped with strain gauges, making it possible to record the forces acting on it.

Recording of the strength of natural ice

In subsequent tests it is planned to perform in-situ measurements. Bending and compression tests will most likely be selected as a relevant determination of the strength parameters. A loading frame has been developed in order to test beams while they are still afloat in-situ.

ARTIFICIAL ICE – PROPERTIES AND MANUFACTURE

General

In connection with research concerning ice forces acting on structures, it was necessary to perform model tests in order to verify various theories. Since cooling facilities were not available at ISVA it became essential to develop an artificial material with properties, at temperatures around 20°C, conforming to the requirements of the relevant model laws.

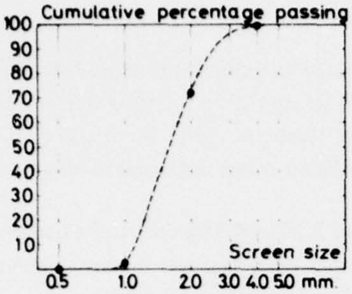

Figure 1. Grading diagram of granulate of plastic described in this report.

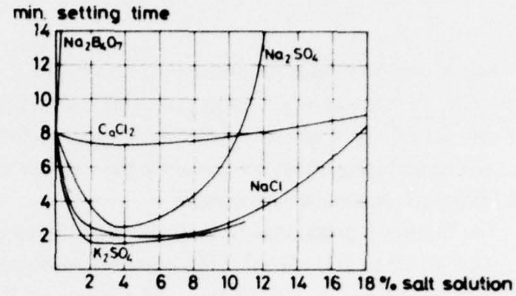

Figure 2. Setting time of plaster of Paris for various salt solutions.

Investigations of floes drifting against structures involve dynamic actions, leading to the Froude model law comprising similitude of inertia forces of water and ice, internal forces, and gravity forces. [Froude number = $u_c/\sqrt{g \cdot e}$, where g is acceleration of gravity (m/s^2), and u_c is the velocity (m/s) of the floe.]

The scale of the models was as follows:

Geometric (model to prototype)	$\lambda = 1_m/1$
Velocity	$\sqrt{\lambda}$
Acceleration	1
Time	$\sqrt{\lambda}$
Force	λ^3
Mass	λ^3
Density	1
Pressure	λ
Strength	λ
Young's modulus	λ
Friction	1 .

With model scales of the order 1/10 to 1/50 this led to a material with a bending strength of 10 to 100 kN/m^2, corresponding to a bending strength of 100 to 5000 kN/m^2 for ice in nature.

Composition of material

The material was composed as follows:

Plaster of Paris ($CaSO_4$)	600 g
Granulate of plastic, 1-2 mm diam (light weight)	80 g
Salt (kitchen salt)	100 g
Borax ($Na_2B_4O_7$)	2 g
Air-entraining agent	10 g
Water	300 g

which yields approximately one liter of material.

It should be noted that the above quantities may require adjustment to compensate for variations in the properties of the plaster of Paris. The plastic granulate was expanded by the Danish firm ANWI from polystyrene supplied by Sinclair-Kopper of Pittsburgh, Pennsylvania. The granulate has a grain diameter between 0.5 and 2.0 mm for ice thicknesses between 1 and 2 cm (see Fig. 1). This can be varied as required.

The unit weight of the granulate is from 40 g/liter up to approximately 80 g/liter. It is possible to re-use the granulate by crushing the sheets and floating the plastic from the plaster of Paris paste. There will be a slight increase in unit weight by this process, which may be taken into account when composing the material.

Salt is used to reduce the strength of the material. A saturated salt solution produces very low strength. Borax acts as a retarder, delaying the setting; a 0.3% solution gives a setting time of about 15 min (at 20°C). Eight minutes is the normal setting time for plaster of Paris. Borax will counteract the accelerating effect produced by the salt (see Fig. 2). The air-entraining agent used is a resin giving approximately 4% air content.

The hardened product saturated with water has a density of 0.90 to 0.95 g/cm^3 and a bending strength of 70 to 200 kN/m^2. The compressive strength is 3 to 5 times as great. Young's modulus is from 10^5 kN/m^2 to 10^6 kN/m^2, and friction $\mu = 0.1$ to 0.3 against a polished plastic plate.

Order of mixing ingredients

The ingredients of the artificial ice material are mixed in the following order:

1. Make the salt solution
2. Add the air-entraining agent
3. Add plaster of Paris
4. Add granulate of plastic.

A small part of the salt solution should be retained for final adjustment of the plasticity of the mix.

RUPTURE THEORY

In some problems involving the destruction of materials it is important to study the type of rupture, which is visible in the form of rupture lines and/or zones. The theory of elasticity possesses the inherent defect of not giving any information about safety against ultimate failure, although in practical engineering there seems to be an increasing tendency to employ design methods which are based on the state of failure. Coulomb's method was probably one of the first to use this principle. However, some problems are too complicated to be solved by use of the theories of elasticity unless very simplifying assumptions are made, which sometimes cause inaccurate results. By using the theory of plasticity it is possible to determine the lines and zones of rupture – in principle. In practice this may present some other problems as exact integration can only be carried out in a few simple cases. However, this can sometimes be overcome by the use of computers.

Dynamic action introduces further complications in establishing the location of rupture lines or zones. Therefore, in the present investigation, elastic as well as plastic theory will be used in order to determine the unknown parameters of the rupture. This is most easily demonstrated as the different formulae are derived.

The basic ideas of this investigation will first be tried on a simplified case of an inclined plane, where solutions can be obtained by use of the theory of elasticity. Then they will be compared with the method based on plasticity and with empirical methods (Reeh 1972, Tryde 1972 and Sørensen 1976).

THEORETICAL APPROACH

Initial contact

When a floe drifts against an inclined wedge (see Fig. 3) initial contact is made at the ridge, where the lower edge of the ice sheet is crushed, while the floe is pushed along the ridge.

A force acts at the point of contact (position I in Fig. 3) and the vertical component of this force will produce a crack propagating along the main axis of the wedge. The force necessary to produce this initial crack can be calculated from the theory of an elastic plate on an elastic support. As the

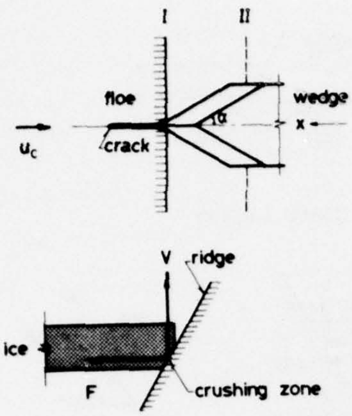

Figure 3. Plan and section of wedge (where α is one-half the angle at the point of the wedge in the horizontal plane, V the vertical force and F the horizontal force, both without friction).

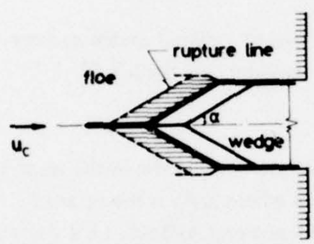

Figure 4. Location of rupture in front of wedge.

floe moves, contact is established along the faces of the wedge, and it can be shown that the area adjacent to the wedge will be a rupture zone. This indicates that rupture will appear randomly, either as crushing along the face or as a combined compression/bending failure at some distance from – and parallel to – the face. During this phase an increasing area of the floe will come in contact with the wedge and the force will gradually increase until the floe engages the full width of the wedge.

By the time the floe has reached position II in Figure 3 (ensuring complete penetration), the cracking pattern will adjust itself to a stage of repeated occurrence. The crack in the centerline will propagate continuously, while the parallelograms in front of the wedge will break off at a given distance from the face (see Fig. 4).

The initial vertical force P_1 required to produce the crack can be found using the theory given by Nevel (1965). Provided the force is acting on a small area of the ice along the ridge:

$$\frac{M_x}{P_1} \cong 1.0$$

where M_x is the unit moment along the crack at the edge and P_1 is the load applied at the edge.

The rupture moment can be written $M_x = r_b W$, where r_b is the bending strength and W is the section modulus. The magnitude of the force will thus be

$$P_1 = \frac{r_b e^2}{6}$$

where e is the thickness of the ice. As an example, r_b = 500 kN/m^2 and e = 0.5 m gives P_1 = 21 kN, which is the initial vertical force necessary to rupture the ice.

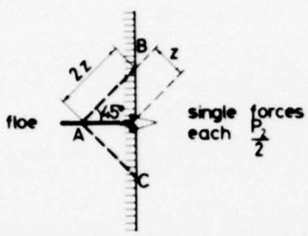

Figure 5. Initial stage of rupture along edge of floe.

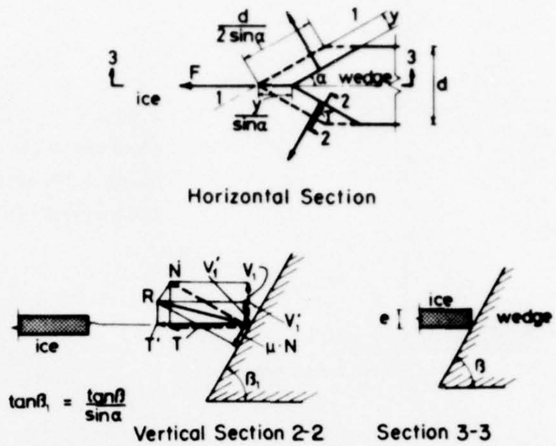

Figure 6. Plan and sections of inclined wedge and acting forces (dotted lines indicate case for $\mu = 0$).

After the formation of the longitudinal crack, the ice along the faces will, in the initial stage, be crushed along the faces of the wedge. This is illustrated in Figure 5 where ½ P_2 is the upward vertical force required to break off each corner. The section modulus along A-B will be $2\,Z\,e^2/6$ where Z is the distance from the point of contact out to the crack. This will give $P_2 = r_b\, e^2\, 2/3$, *i.e. a force which is four times as great as* P_1, *both sides breaking off simultaneously.*

This situation only prevails during a short period before the parallelogrammic pattern is initiated. The force is independent of the width of the wedge, and for all practical purposes it is rather small, being only a small percentage of the total force for which the structure must be designed.

Forces acting on wedge

The geometry and the principal force diagrams for forces acting on a wedge are illustrated in Figure 6. In this figure N is the force normal to the face, T' is the force in the horizontal plane, V_1' is the vertical force on the face and β is the inclination of the wedge to horizontal. The coefficient of friction is denoted as μ, and all forces with a prime mark indicate that friction has been included. The forces can be expressed as follows:

$$T' = V_1 \left(\frac{\tan\beta}{\sin\alpha} + \mu \right) = V_1 C_2$$

where

$$C_2 = \frac{\tan\beta}{\sin\alpha} + \mu$$

$$F = 2T' \sin\alpha = 2\, V_1\, C_2 \sin\alpha$$

$$V_1' = V_1 \left(1 - \mu \frac{\tan\beta}{\sin\alpha} \right) = V_1\, C_1$$

where

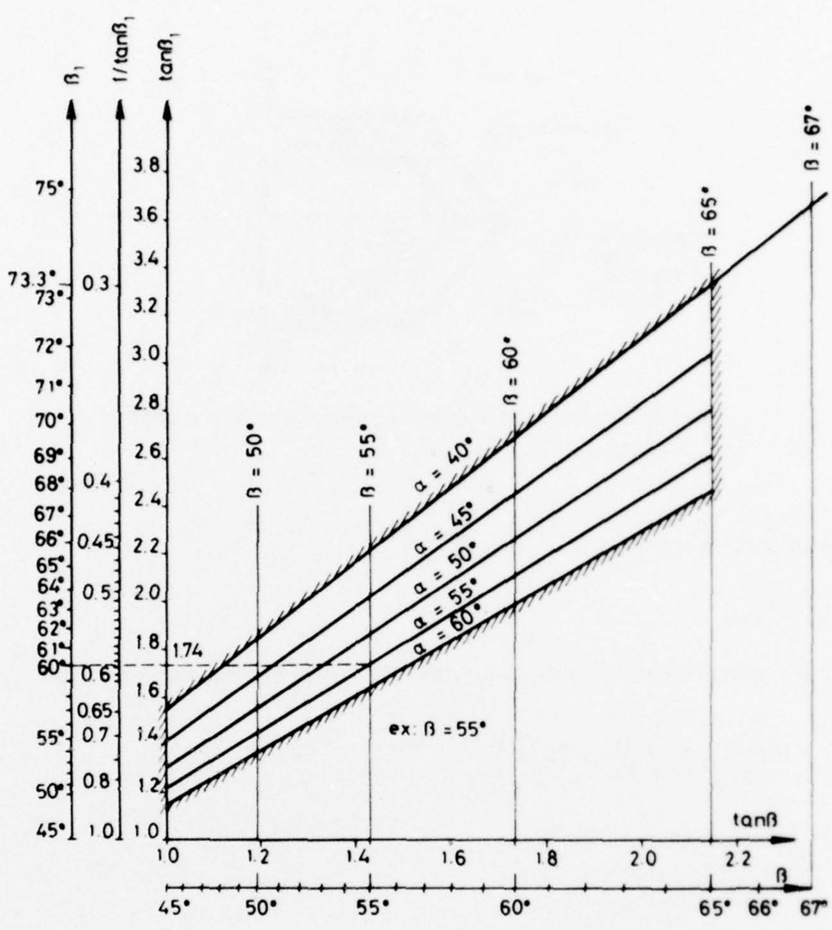

Figure 7. Inclined wedge, parameters α, β, β_1, $\tan\beta$, $\tan\beta_1 = \tan\beta/\sin\alpha$.

$$C_1 = 1 - \mu \frac{\tan\beta}{\sin\alpha}$$

and V_1 is the vertical force on each face in case of no friction. The functional relationship of the various parameters is shown in Figure 7.

Friction in the horizontal plane has been neglected, as the influence of this on the rupture is negligible. The friction in the vertical direction may possibly be increased by ice being frozen to the face of the wedge (thus preventing the ice from moving upwards). However, as the rupture patterns will hardly be affected, the increase of the total force will, for $\mu = 0.1$, amount to approximately 10%.

Maximum force and actual force

The maximum force F_{max} on a given section in front of the structure is

$$F_{max} = r_c e d$$

where r_c = compression strength (kN/m^2)
e = thickness of the ice (m)
d = width of the structure (m).

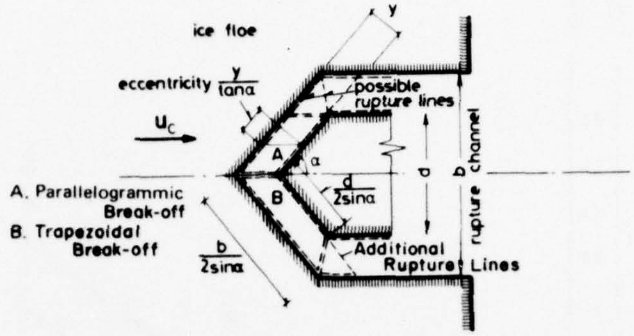

Figure 8. Rupture lines in front of inclined wedge (b = width of rupture channel).

The actual force is expressed as

$$F = C_F\, F_{max}$$

where C_F is a reduction factor, which is a function of a system of nondimensional data:

$$C_F = \phi\left(\frac{E}{\rho u_c^2}, \frac{e}{d}, \frac{y}{e}, \frac{r_b}{r_c}, \alpha, \beta, \mu\right)$$

where E = Young's modulus (kN/m²)
ρ = density of ice (tonnes/m³)
u_c = velocity of the floe (m/s)
y = characteristic breakoff distance (m)
r_b = flexural strength (kN/m²)
α = included angle at the point of the wedge in the horizontal plane
β = inclination of wedge to horizontal
μ = coefficient of friction.

Principal stress equations

The assumed rupture lines are indicated in Figure 8.

The load along the face on each side of the wedge is assumed to be uniformly distributed. The vertical and the horizontal forces produce an eccentric load combination on the rupture plane in the ice, which results in the following maximum boundary compressive stress:

$$\sigma_{bc} = \frac{T'}{e\,(b/2\sin\alpha)} + \frac{V_1'y - T'x}{{}^1/_6\, e^2\,(b/2\sin\alpha)} + \frac{T'\,(k_1 y/\tan\alpha)}{{}^1/_6\,(b/2\sin\alpha)^2\, e} \qquad (1)$$

where y is the breakoff distance as shown in Figure 8, and x is the eccentricity of the horizontal force, which can be expressed as $x = fe$, f being the eccentricity coefficient. This coefficient can be written as $f = \frac{1}{2}(1 - C_F)$. The eccentricity in the horizontal plane would be $y/\tan\alpha$ if the width of the rupture channel were equal to the width of the wedge. As additional cracks develop as shown in Figure 8, it is proposed to adjust the eccentricity to a value between the two extremes illustrated in Figure 8. Therefore, the factor k_1 has been introduced.

For the section modulus $^1/_6\,(b/2\sin\alpha)^2\,e$, a modified expression $1.5\cdot{}^1/_6\,[dbe/(2\sin\alpha)^2]$ will be used in order to simplify the following formula:

$$\sigma_{bc}=\frac{V_1C_2\,2\sin\alpha}{eb}+\frac{(V_1C_1y-V_1C_2x)\,12\sin\alpha}{e^2b}+V_1C_2\frac{k_1y\,24\sin^2\alpha}{1.5bde\tan\alpha} \tag{2}$$

using the forces derived in the section *Forces Acting on Wedge.*

By inserting $x=(e/2)(1-C_F)$, eq 2 can be written as

$$\sigma_{bc}=\frac{2V_1\sin\alpha}{eb}\left\{C_2\,[1-3(1-C_F)]+\frac{y}{e}\,6C_1+\frac{y}{e}\,C_2\,8k_1\frac{e}{d}\,\cos\alpha\right\}. \tag{3}$$

Introducing the system constant of $C_5=6C_1+6C_2\,(e/d)\cos\alpha$ and a value of $k_1=0.75$, eq 3 can be written as

$$\sigma_{bc}=\frac{2V_1\sin\alpha}{eb}\left\{C_2\,(3C_F-2)+\frac{y}{e}\,C_5\right\}. \tag{4}$$

By using $F'=2\,V_1C_2\sin\alpha$:

$$V_1=F'\,\frac{1}{2\sin\alpha\,C_2}$$

$$C_F=\frac{F'}{F_{max}}=\frac{F'}{r_ced}=\frac{r_{bc}ebC_2}{r_ced\,[C_2\,(3C_F-2)+(y/e)\,C_5]}\,. \tag{5}$$

The rupture strength r_{bc} is a function of ϵ (the r_b/r_c ratio) and the reduction factor C_F (see Fig. 9). It can be shown that the rupture strength for a combined axial load and bending moment will be a tension failure in the interval $0<C_F<(1-\epsilon/2)$ giving

$$\frac{r_{bc}}{r_c}=2C_F+\epsilon \tag{6}$$

and a compression failure in the interval $1-\epsilon/2<C_F<1.0$ giving

$$\frac{r_{bc}}{r_c}=1\,. \tag{7}$$

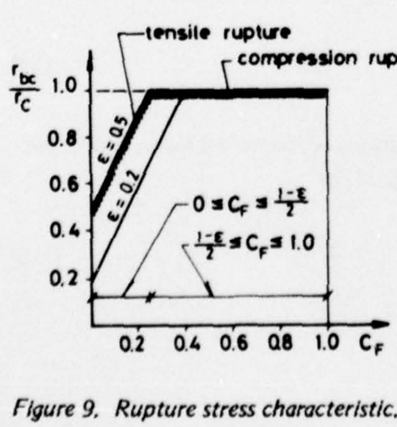

Figure 9. Rupture stress characteristic.

For the width b of the rupture channel the following empirical formula is suggested, where the ratio b/d is a function of C_F:

$$\frac{b}{d}=-1.5\,C_F+2.5 \tag{8}$$

which is based on experimental data. It is correct for $C_F=1$, corresponding to the case with vertical faces. Equation 5 will give, when inserting eq 6, 7 and 8:

$$C_F = \frac{-3C_F^2 + 5C_F - 1.5\epsilon C_F + 1.5_\epsilon + \epsilon}{3C_F - 2 + (y/e)(C_5/C_2)} \tag{9}$$

or as an expression of y/e for interval 1 $\left(0 < C_F < \frac{1-\epsilon}{2}\right)$

$$\left(\frac{y}{e}\right)_1 = \frac{-6C_F + 7 - 1.5\epsilon + (2.5\epsilon/C_F)}{C_5/C_2} \tag{10}$$

and

$$C_F = \frac{-1.5C_F + 2.5}{3C_F - 2 + (y/e)(C_5/C_2)}$$

or as an expression of y/e for interval 2 $\left(\frac{1-\epsilon}{2} < C_F < 1.0\right)$

$$\left(\frac{y}{e}\right)_2 = \frac{-3C_F + 0.5 + (2.5/C_F)}{(C_5/C_2)} \tag{11}$$

Two unknowns appear in this equation, a solution requiring that one more equation be found. This is done further below, where the dynamic equation is derived for the ice pieces breaking off in front of the wedge.

Equation 10 may be used to determine the value of y/e in the case of a floe at rest in front of the wedge, as we then know the values of the force F equal to the wind shear or the water shear acting on the floe, and F_{max} is also known. Thus

Wind shear: $\tau_i = 4.8 \cdot 10^{-3} \cdot \frac{1}{2}\rho_a\, 10^{-6}\, U^2_{wind}$ kN/m^2

Water shear: $\tau_w = 5.4 \cdot 10^{-3} \cdot \frac{1}{2}\rho_w\, 10^{-3}\, U^2_{water}$ kN/m^2

where U_{wind} and U_{water} are the wind and water velocities, respectively. For $F = r_c ed$ we obtain $C_F = 1$. For $F > r_c ed$ the floe will move continuously.

To determine the values of C_1/C_2 and $C_5/C_2 \approx C_3$ use Figures 7 and 10.

Dynamic equations

The equation of angular momentum of the ice piece broken off can be written as

$$\frac{1}{2}(y V_1' - T'x)dt = I_A d\omega \tag{12}$$

where I_A is the moment of inertia about A-A, x the eccentricity of horizontal force, and ω the angular velocity. The vertical force can be written (see Fig. 11) as

$$V_1' = \frac{r_c d}{2\sin\alpha}\ z_h \tag{13}$$

which gives

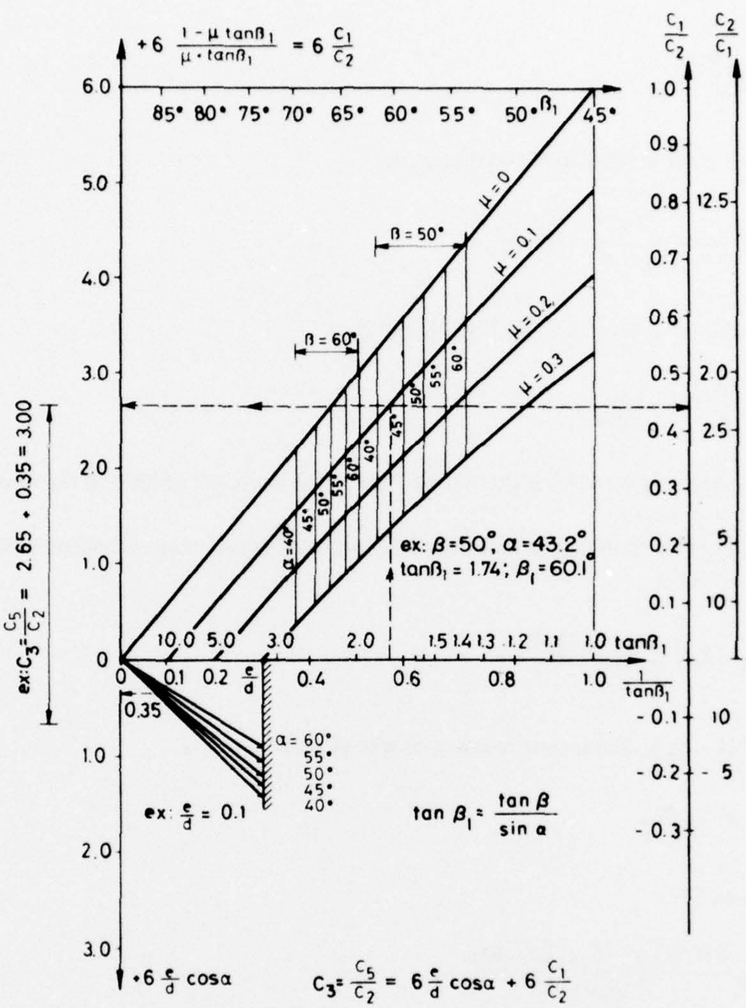

Figure 10. Inclined wedge, values of C_1/C_2 and C_5/C_2 for different values of α, β and μ.

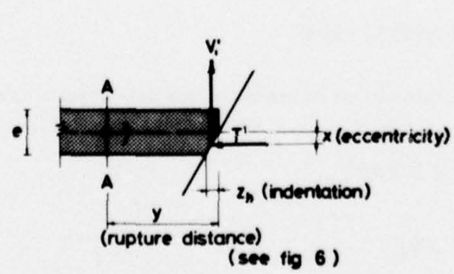

Figure 11. Forces acting at contact area.

$$z_h = \frac{V_1' 2 \sin\alpha}{r_c d} \quad . \tag{14}$$

The time required for making an indentation z_h is

$$dt = \frac{z_h}{u_c \sin\alpha} = \frac{2V_1'}{u_c r_c d} = \frac{2V_1 C_1}{u_c r_c d} \quad . \tag{15}$$

The moment of inertia about A-A (in Fig. 11) will be

$$I_A = C_A \frac{1}{3} y^3 e \frac{d\rho}{2 \sin\alpha} \quad . \tag{16}$$

Actually the moment of inertia is slightly larger, due to the increased width b of the rupture channel. C_A will be explained below.

From the theory of elasticity, at time dt the deflection of the ice sheet measured vertically at the point of contact is

$$f_A = C_B \frac{V_1 (C_1 y - C_2 ef) y^2}{3EI_s} \tag{17}$$

where $f = 1/2\,(1 - C_F)$. The second moment of area of section A-A is

$$I_s = \frac{1}{12} e^3 \frac{b}{2 \sin\alpha} \quad . \tag{18}$$

Inserting I_s in eq 17:

$$f_A = C_B \frac{8V_1 (C_1 y - C_2 ef) y^2 \sin\alpha}{Ee^3 b} \quad . \tag{19}$$

C_B will be explained below. The effect of the horizontal force on the deflection is not exactly as stated, but the error is small and is justified by the simplification obtained. C_A was introduced in eq 17 to allow for the effect of the virtual mass of the water below the ice (Engelund 1966).

The virtual mass effect may be derived as follows. For a strip 1 m wide the virtual moment of inertia I_v is

$$I_v \sim 0.2\rho y^4 \quad \text{(from Engelund 1966).} \tag{20}$$

If it is assumed that $y \sim 3e$ the moment of inertia of the strip of ice rotated would be approximately

$$I_i \cong \frac{1}{3} \rho y^3 e = \frac{1}{9} \rho y^4 \sim 0.1 \rho y^4 \tag{21}$$

$$I_{v+i} = I_v + I_i = 0.3\rho y^4 = 3I_i \quad . \tag{22}$$

Actually the resulting moment of inertia is a function of y/e, but for simplification we shall use a factor $C_A = 3$ for the moment of inertia of the ice piece itself in order to obtain the total moment of inertia.

If full fixity is assumed at the rupture line (A–A), the value of C_B will be 1.0. If, on the other hand, one assumes that the action is that of a continuous beam with the next support at a distance $4y$ from the line of rupture, the deflection will be five times larger, giving $C_B = 5.0$, which is a realistic assumption. The effect of these values is not as great as it would seem at first sight, as they appear in the 1/4 power in the final formula.

The instantaneous change of ω may be expressed as

$$d\omega = \frac{f_A}{dty} = C_B \frac{4(C_1 y - C_2 ef) y u_c r_c \sin\alpha d}{Ee^3 C_1 b} \quad . \tag{23}$$

Inserting eq 15 and eq 19 V_1 is

$$V_1 = 0.82\sqrt{C_A C_B} \sqrt{\frac{\rho}{E}} \frac{1}{C_1} u_c r_c \frac{y^2}{e} \sqrt{\frac{d^3}{b}} \, . \tag{24}$$

Inserting in

$$\sigma_{bc} = \frac{2V_1 \sin\alpha}{eb} \left| C_2(3C_F - 2) + \frac{y}{e} C_5 \right| \tag{25}$$

$$\frac{\sigma_{bc}}{1.64\sqrt{C_A C_B} \sqrt{\rho/E} \; (C_2/C_1)\, r_c u_c \sin\alpha} = \sqrt{\left(\frac{d}{b}\right)^3} \left| \left(\frac{y}{e}\right)^2 (3C_F - 2) + \frac{C_5}{C_2} \left(\frac{y}{e}\right)^3 \right|$$

by using

$$\frac{r_{bc}}{r_c} = 2C_F + \epsilon \qquad \text{in interval 1} \left(0 < C_F < \frac{1-\epsilon}{2}\right), \tag{26}$$

and $$\frac{r_{bc}}{r_c} = 1 \qquad \text{in interval 2} \left(\frac{1-\epsilon}{2} < C_F < 1\right).$$

Furthermore, if $b/d = -1.5\,C_F + 2.5$ the following are found:

interval 1

$$0.61 \sqrt{\frac{1}{C_A C_B}} \sqrt{\frac{E}{\rho u_c^2}} \frac{1}{\sin\alpha} \frac{C_1}{C_2}$$

$$= \frac{1/\sqrt{(-1.5\,C_F + 2.5)^3}}{2C_F + \epsilon} \left| \left(\frac{y}{e}\right)_1^2 (3C_F - 2) + \frac{C_5}{C_2} \left(\frac{y}{e}\right)_1^3 \right| \tag{27}$$

interval 2

$$0.61 \sqrt{\frac{1}{C_A C_B}} \sqrt{\frac{E}{\rho u_c^2}} \frac{1}{\sin\alpha} \frac{C_1}{C_2}$$

$$= \frac{1}{\sqrt{(-1.5C_F + 2.5)^3}} \left| \left(\frac{y}{e}\right)_2^2 (3C_F - 2) + \frac{C_5}{C_2} \left(\frac{y}{e}\right)_2^3 \right| \tag{28}$$

in which eq 10 and 11 should be inserted. When applying $C_A = 3$ and $C_B = 5$ in eq 16 and 19, a system parameter C can be introduced for intervals 1 and 2, respectively:

$$C = 0.16 \sqrt{\frac{E}{\rho u_c^2 \sin^2\alpha}} \; \frac{C_1}{C_2} \left(\frac{C_5}{C_2}\right)^2 \tag{29}$$

$$C = \frac{1/\sqrt{(-1.5C_F + 2.5)^3}}{2C_F + \epsilon} \left\{\left(\frac{y}{e}\right)_1^2 \left(\frac{C_5}{C_2}\right)^2 (3C_F - 2) + \left(\frac{y}{e}\right)_1^3 \left(\frac{C_5}{C_2}\right)^3\right\} \tag{30}$$

and

$$C = \frac{1}{\sqrt{(-1.5C_F + 2.5)^3}} \left\{\left(\frac{y}{e}\right)_2^2 \left(\frac{C_5}{C_2}\right)^2 (3C_F - 2) + \left(\frac{y}{e}\right)_2^3 \left(\frac{C_5}{C_2}\right)^3\right\} \tag{31}$$

Intermittent nature of force

The acting forces are of an intermittent nature since they increase abruptly each time a parallelogrammic piece is about to break off, after which the forces decrease until the next encounter takes place. The period of time between each peak can be expressed by

$$t_c = \frac{y}{u_c \sin\alpha} = \left(\frac{y}{e}\right) \frac{e}{u_c \sin\alpha} \tag{32}$$

where y/e can be taken from eq 10 and 11. Each peak of the force takes place within 1/5 to 1/50 of a second.

Theoretical curve

The curves illustrating the functions in the two intervals are shown in Figure 12. The results of model tests, which will be described later, have also been plotted (see *Plotting of Results* below). The dispersion of results is rather noticeable, but the agreement is fairly good.

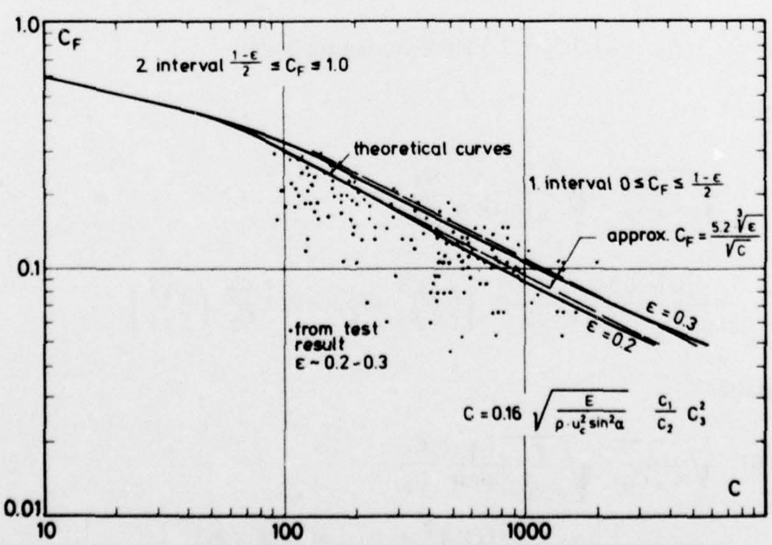

Figure 12. Theoretical curves — test results.

An illustrative approximation of these theoretical curves can be expressed as

$$C_F = \frac{5.2\sqrt[3]{\epsilon}}{\sqrt{C}} \qquad \text{for interval 1} \left(0 < C_F < \frac{1-\epsilon}{2}\right) \tag{33}$$

VERIFICATION OF THEORY BY MODEL TESTS

Description

In order to verify the theoretical investigation, model tests using the artificial material described in *Artificial ice – Properties and Manufacture* were performed.

A flume 2 m wide with a water depth of 0.3 m was used. The experimental floe, with an area of approximately **8** m^2, consisted of plywood, except for an area of 1.2×2.0 m, where the artificial material was used (see Fig. 13).

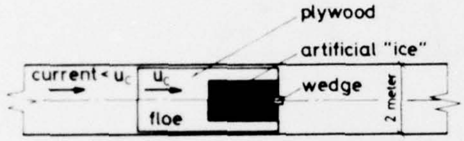

Figure 13. Plan of flume.

An inclined wedge 0.16 m wide, with $\alpha = 50°$ and $\beta = 55°$, was used as the object. The whole wedge could be rotated in order to test various values of β. The wedge was made of smooth Perspex in order to minimize friction. A dynamometer with strain gauges was placed inside the wedge, making it possible to record instantaneous horizontal/vertical forces and the moment. These force and moment signals as well as a velocity signal in the form of pulses from a photo cell beam intercepted by a "comb" placed on the floe were recorded on a high speed (30-in./s) tape recorder.

For visual observation of the signal, the tape recorder was played back at slow speed (15/32 in./s) and a 5-channel pen recorder was used to obtain curves from all recording components (see Fig. 14).

Tests were made with varying velocities. For detailed information concerning test results, see App. A.

Width of rupture channel

Test results show that the width of the rupture channel is considerably larger than the width of the wedge, in some cases about 2.5 times as great. It has not been possible to develop a theory that can predict the rupture pattern. Therefore, the width is given by the empirical formula $b/d = -1.5C_F + 2.5$, which gives fairly good agreement with the tests.

Plotting of results

The moment and horizontal and vertical forces were recorded during the breakoff of each of the parallelogrammic pieces. The C_F coefficient is determined from

$$C_F = \frac{F}{F_{max}}$$

where

$$F_{max} = r_c e d,$$

which is known for each test, as the compression strength and thickness of the ice are measured. Furthermore, the velocity of the floe was recorded and the size of the breakoff measured.

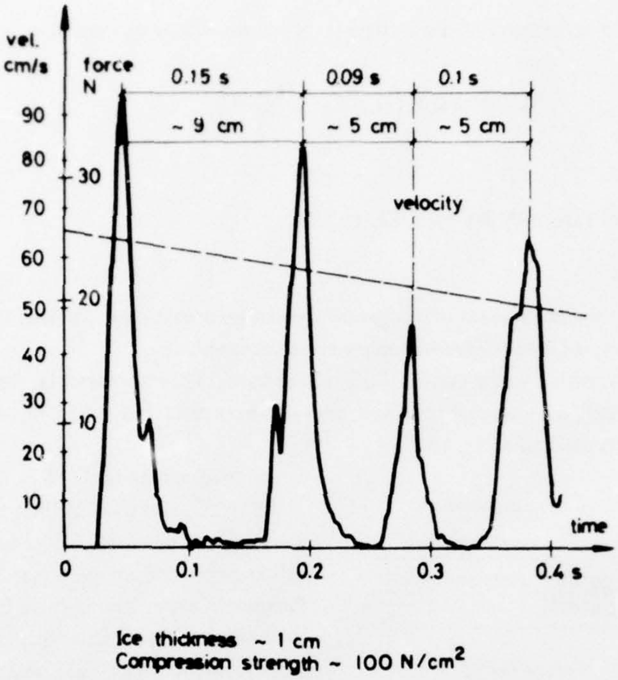

Figure 14. Typical force variation.

Each test comprised about 20 encounters. Based on the magnitude of the vertical and horizontal forces and the geometry of the wedge, it was possible to determine the friction coefficient. The variation of the friction was usually not appreciable; therefore, a mean value was adopted. The system data C_1/C_2, C_5/C_2, C were determined for each test. At the first encounter, only where the floe had been precut with straight lines to fit the wedge plan profile was it possible to obtain a simultaneously symmetrical breakoff. In no other encounter during the tests had this been experienced. It was therefore concluded that symmetrical occurrence is rare.

Depending on the regularity of the rupture line, which is a straight line only in theory, the rising time of the load is approximately 1/100 s. During that time the floe drifting with a velocity of, for example 50 cm/s, would move 0.5 cm, which is the size of the deviation from the straight line of the rupture. Therefore, it is likely that the two peaks, on each side of the wedge, will occur in a staggered pattern. This implies that the recorded C_F value from the test should be multiplied by a mean value of 1.3 before being plotted. In Figure 12, the test results have been plotted accordingly, so as to compare them with the theory based on simultaneous breakoff.

As shown in the recordings of the forces, the rupture force shows a clear regularity although the rupture pattern itself may look rather irregular. The forces acting are of an intermittent nature, since they increase abruptly each time a parallelogrammic piece is about to break off, after which the forces decrease until the next encounter takes place. Usually, it is observed that the rupture is asymmetrical, as the ice pieces on each side of the wedge do not break off simultaneously but rather in rapid succession after each other (see Fig. 14).

A few tests have been performed with natural ice at Hamburgische Schiffbau-Versuchsanstalt. The results are in fair agreement with the theory, although the ruptures apparently are slightly different from those observed with artificial ice. The natural ice was, however, weaker than the artificial material used in Copenhagen. It appears that the parallelogrammic pieces were broken up into smaller parts at Hamburg, but whether this took place simultaneously was difficult to observe. This should be further studied.

ENGINEERING APPLICATION

The theory presented in the preceding pages is impractical for engineering calculation, because the formulae are complicated and lack a clear presentation of the functional relationship of the various system constants. It is therefore recommended to use the formula:

$$C_F = \frac{5.2\sqrt[3]{r_b/r_c}}{\sqrt{C}} .$$

The forces acting are, as stated, of an intermittent nature. The period of time between each peak force can be expressed as

$$t_c = \frac{y}{u_c \sin\alpha} = \left(\frac{y}{e}\right)\frac{e}{u_c \sin\alpha} \sim \frac{1.3}{C_5/C_2}\sqrt[3]{C}\ \frac{e}{u_c \sin\alpha}$$

where an approximate equation $y/e \sim 1.3/(C_5/C_2)\sqrt[3]{C}$ has been used. This should be considered in the design of the structures, as resonance response can prove to be catastrophic. An example of the use of this method would be the following:

$E = 5 \times 10^6$ kN/m^2	$\rho = 0.93$ t/m^3
$u_c = 1.0$ m/s	$e/d = 0.1$
$\alpha = 45°$	$\beta = 60°$
$\mu = 0.1$	$\epsilon = 0.2$

$$C = 0.16\sqrt{\frac{E}{\rho u_c^2}}\ \frac{1}{\sin\alpha}\ \frac{C_1}{C_2}\ C_3^2$$

$$C = 0.16\sqrt{\frac{5 \cdot 10^6}{0.93 \cdot 1.0^2}}\ \frac{1}{\sin\alpha}\ \frac{C_1}{C_2}\ C_3^2 = 371\ \frac{1}{\sin\alpha}\ \frac{C_1}{C_2}\ C_3^2$$

$$C_1 = 1 - \mu\frac{\tan\beta}{\sin\alpha} = 1 - 0.1\ \frac{1.732}{0.707} = 0.755$$

$$C_2 = \mu + \frac{\tan\beta}{\sin\alpha} = 0.1 + \frac{1.732}{0.707} = 2.549$$

$$\frac{C_1}{C_2} = 0.296$$

$$\frac{C_5}{C_2} = C_3 = 6\frac{C_1}{C_2} + 6\frac{e}{d}\cos\alpha = 1.777 + 0.424 = 2.201$$

$$C = 371\ \frac{1}{0.707} \cdot 0.296 \cdot 2.2^2 = 751.8.$$

Engineering formula (eq 33)

$$C_F = \frac{5.2\sqrt[3]{\epsilon}}{\sqrt{C}} = \frac{5.2\sqrt[3]{0.2}}{\sqrt{751.8}} = 0.11.$$

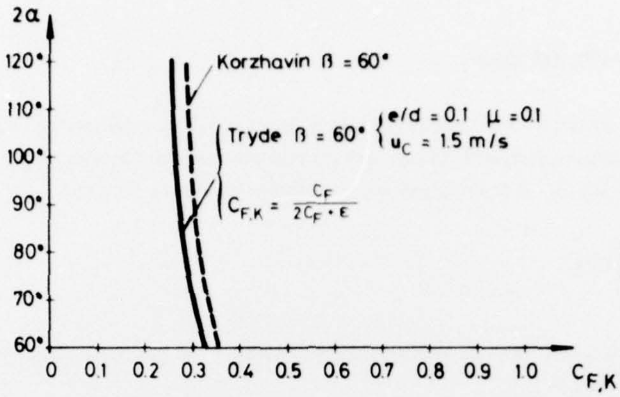

Figure 15. Comparison with Korzhavin's results.

COMPARISON WITH KORZHAVIN'S RESULTS

In Korzhavin (1971, Table 34, p. 165) he has given the S_0 values for the reduction coefficients. In order to compare the S_0 values with results given in this paper, they have been multiplied with $\tan\beta$, using formula 11.30 in Korzhavin's report.

The results have been plotted in Figure 15.

Detailed information concerning the system constants used by Korzhavin is not available. The following constants have been used in the curve for $\beta = 60°$: $E = 5 \times 10^6$ kN/m^2, $\rho = 0.93$ tonne/m^3, $u_c = 1.5$ m/s, $e/d = 0.1$, $\mu = 0.1$, and $\epsilon = 0.2$.

The reduction coefficient C_F has been transformed to $C_{F,K} = C_F/(2C_F + 0.2)$ as Korzhavin is using the bending strength in his formula. As can be seen in Figure 15, the plotted curve agrees fairly well with the results of Korzhavin.

RECOMMENDATIONS FOR FUTURE RESEARCH

In a work of this kind there will always be some unsolved problems which require further study.

It has been possible to develop a theory whereby the forces can be predicted with an acceptable accuracy, and model tests have been made in order to verify the theory. However, certain problems are not solved, and therefore it has been necessary to use empirical solutions in some cases. The width of the rupture channel has not been theoretically determined, and further developments should make this possible. The combined axial and bending failure also needs further investigation. The r_b/r_c ratio should be reevaluated as this is quite dependent on the method of performing compression tests (the IAHR ice committee is at present studying this problem).

The hydrodynamic mass of the ice debris should be further evaluated by model tests. The rupture pattern and the irregularity of the rupture lines should also be examined as this has an influence on the dynamic forces. Finally, it is emphasized that model tests with artificial as well as natural ice should be continued in order to clarify the remaining unsolved problems in this paper.

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APPENDIX A. TEST DATA

Introduction

Some of the test results are given in the following tables. The bending strength r_b and the compression strength r_c of the ice varied from test to test, but in this presentation the strength parameters have all been normalized to a mean value using $\epsilon = 0.22$. All the force registrations have been corrected accordingly. The rupture distance y is determined from $y = x \sin\alpha$, where x is measured in the direction of the motion.

Detailed information from the test is available in an ISVA internal report by C. Sørensen (1974).

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Test with artificial ice: No. 3/5.1

Type: Inclined wedge $\alpha = 50^o$ $\beta = 55^o$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.904$ g/cm^3

$e = 1.1$ cm $\mu = 0.22$ $C_1/C_2 = 0.287$ $C_5/C_2 = 1.986$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{154.7}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc. coef. C_F	Notes
1	152	37.7	16.2	0.18	75	15	3	26	0.24	
2	158	20.1	7.6	0.21	73	18	6	25	0.13	
3	158	29.7	8.7	0.23	72	24	8	35	0.16	
4	164	26.1	11.3	0.17	69	32	14	35	0.17	
5	195	22.5	9.0	0.20	59	46	9	25	0.15	
6	205	24.2	8.3	0.24	55	55	4	25	0.16	
7	221	8.4	3.9	0.15	52	59	4	25	0.054	
8	235	17.6	5.3	0.27	48.5	63	5	28	0.11	
9	266	16.8	6.5	0.21	43	68	4	28	0.11	
10	277	6.8	2.5	0.22	41	72	3	16	0.044	
11	292	10.9	4.0	0.22	39	75	4	25	0.070	
12	308	15.6	5.7	0.23	37	79	5	25	0.101	
13	338	15.1	5.7	0.22	34	84	4	25	0.098	
14	400	9.9	3.6	0.23	28	88	4	26	0.064	
15	430	13.2	4.9	0.22	26	92	5	26	0.085	
16	472	18.7	7.0	0.22	24	97	12	35	0.12	
17	718	15.7	5.0	0.26	16	109	6	26	0.10	

Test with artificial ice: No. 3/5.2

Type: Inclined wedge $\alpha = 50^o$ $\beta = 55^o$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.904$ g/cm^3

$e = 1.1$ cm $\mu = 0.24$ $C_1/C_2 = 0.263$ $C_5/C_2 = 1.844$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{154.7}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc. coef. C_F	Notes
1	190	19.1	7.9	0.19	47.5	15	4	18	0.12	
2	199	26.4	8.8	0.25	45.5	19	9	25	0.17	
3	225	22.6	7.0	0.26	40.8	28	9	26	0.15	
4	283	24.6	8.0	0.25	32.0	37	6	28	0.16	
5	539	6.0	1.9	0.26	16.0	43	5	29	0.04	?
6	729	5.6	1.8	0.26	12.5	48	5	29	0.04	?

Test with artificial ice: No. 3/6.1

Type: Inclined wedge $\alpha = 48.1^o$ $\beta = 50^o$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.901$ g/cm^3

$e = 1.0$ cm $\mu = 0.21$ $C_2/C_1 = 0.37$ $C_5/C_2 = 2.48$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{140.6}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc. coef. C_F	Notes
1	337	21.3	10.1	0.22	71	20	4	31	0.15	high
2	345	13.0	6.7	0.19	70	24	4	16	0.10	
3	352	19.8	10.5	0.18	68	28	4	30	0.14	high
4	376	16.4	7.8	0.22	64	39	11	24	0.12	
5	384	16.9	7.7	0.23	62	43	6	24	0.12	
6	400	6.2	4.5	0.06	60	49	7	32	0.044	low
7	416	9.6	4.6	0.22	58	56	2	26	0.069	low
8	424	7.7	4.7	0.13	57	58	5	25	0.055	low
9	441	18.6	9.0	0.21	55	63	10	33	0.132	high
10	488	13.2	6.1	0.23	49	73	9	30	0.094	
11	536	13.5	6.5	0.22	45	82	5	20	0.096	
12	568	10.1	4.7	0.23	42	87	2	20	0.072	
13	600	13.7	6.1	0.24	40	89	5	22	0.098	
14	632	9.1	4.3	0.22	38	94	4	25	0.065	
15	705	13.0	5.6	0.25	34	98	5	18	0.093	
16	824	13.9	6.7	0.22	29	103	7	20	0.099	
17	880	10.6	6.0	0.16	27	110	5	20	0.076	
18	1040	7.4	4.0	0.18	23	115	5	24	0.053	low
19	1321	6.2	3.4	0.17	18	119	4	24	0.044	low

Test with artificial ice: No. 3/6.2

Type: Inclined wedge $\alpha = 48.1^o$ $\beta = 50^o$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.901$ g/cm^3

$e = 1.0$ cm $\mu = 0.20$ $C_1/C_2 = 0.374$ $C_5/C_2 = 2.5$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{140.6}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist.x cm	W. of rupt.channel in cm	Reduc. coef. C_F	Notes
1	454	16.9	9.2	0.17	58	24	7	16	0.12	
2	478	11.5	5.6	0.21	52	31	5	24	0.082	
3	536	9.6	5.1	0.18	47	36	2	18	0.060	
4	551	13.5	6.7	0.21	45	38	5	16	0.096	
5	673	9.1	4.7	0.19	37	43	4	24	0.060	
6	779	7.2	3.8	0.19	32	47	1	18	0.061	
7	892	10.6	5.1	0.22	28	48	5	18	0.078	
8	1305	11.5	5.0	0.25	19	53	5	24	0.082	
9	1912	10.1	4.7	0.23	13	58	5	20	0.072	

Test with artificial ice: No. 3/7.1

Type: Inclined wedge $\alpha = 48.1^{o}$ $\beta = 50^{o}$ d = 16 cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.916$ g/cm^3

e = 0.9 cm $\mu = 0.20$ $C_1/C_2 = 0.382$ $C_5/C_2 = 2.516$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{126.58}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal Force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σx cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc. coef. C_F	Notes
1	385	33.0	16.3	0.21	66	20	7	32	0.26	high
2	507	11.8	5.9	0.20	63	27	4	16	0.093	
3	412	14.2	7.8	0.17	60.5	31	6	30	0.113	
4	428	14.0	6.7	0.22	58	37	4	30	0.11	
5	452	10.0	5.0	0.20	55	41	3	22	0.079	
6	477	17.7	7.8	0.25	53	44	9	28	0.14	
7	534	10.6	5.7	0.18	47.5	53	5	20	0.084	
8	576	11.3	5.6	0.21	44	58	6	30	0.089	
9	633	10.6	5.8	0.17	40	64	7	30	0.084	
10	732	10.6	5.3	0.21	36	71	5	27	0.084	
11	905	8.9	5.1	0.15	28.2	76	6	27	0.070	
12	988	10.5	5.3	0.20	25.8	82	5	24	0.083	
13	1210	9.2	4.5	0.21	21	87	7	35	0.073	
14	1687	3.2	2.1	0.10	14.8	94	7	40	0.025	low

Test with artificial ice: No. 3/7.2

Type: Inclined wedge $\alpha = 48.1^{o}$ $\beta = 50^{o}$ d = 16 cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.918$ g/cm^3

e = 0.9 cm $\mu = 0.21$ $C_1/C_2 = 0.371$ $C_5/C_2 = 2.45$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{126.58}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc. coef. C_F	Notes
1	454	10.7	5.3	0.21	52	7	7	29	0.085	
2	469	4.4	2.4	0.17	50	14	5	24	0.035	
3	485	7.6	3.6	0.22	48	19	5	33	0.060	
4	508	10.6	4.9	0.23	46	24	4	18	0.084	
5	524	12.3	6.4	0.19	44.5	28	4	22	0.097	
6	579	8.9	4.4	0.21	40	32	7	22	0.070	
7	664	10.5	5.6	0.18	35	39	2	22	0.083	
8	705	13.4	6.9	0.19	33	41	8	28	0.106	
9	806	10.4	4.9	0.22	28.8	49	6	20	0.082	
10	1017	11.6	5.7	0.21	23	55	5	25	0.092	

Test with artificial ice: No. 3/8.1

Type: Inclined wedge $\alpha = 48.1^o$ $\beta = 50^o$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.890$ g/cm^3

$e = 1.0$ cm $\mu = 0.19$ $C_1/C_2 = 0.384$ $C_5/C_2 = 2.56$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{140.6}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc. coef. C_F	Notes
1	493	29.2	17.1	0.15	54.5	25	5	22	0.208	Peak-loading time ~1/100 sec.
2	519	12.3	6.2	0.20	52.5	30	8	22	0.088	
3	603	15.4	6.9	0.24	45.0	38	4	34	0.110	
4	680	10.6	5.6	0.19	40.0	42	3	32	0.075	
5	714	7.2	4.7	0.10	38.0	45	3	28	0.051	Low friction
6	748	7.2	4.3	0.14	36.5	48	3	28	0.051	
7	773	11.0	5.3	0.22	35.0	51	4	28	0.078	
8	901	13.1	6.0	0.23	30.0	55	7	28	0.093	
9	1079	8.4	4.6	0.18	25.0	62	5	30	0.060	
10	1360	6.8	3.2	0.22	20.0	67	3	29	0.048	
11	1521	6.8	3.3	0.21	17.8	70	3	27	0.048	

Test with artificial ice: No. 3/9.1

Type: Inclined wedge $\alpha = 51.6^o$ $\beta = 60^o$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.91$ g/cm^3

$e = 1.0$ cm $\mu = 0.17$ $C_1/C_2 = 0.26$ $C_5/C_2 = 1.79$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{140.64}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σx cm	Break off dist. x cm	W. of rupt.channel in cm	Reduc. coef. C_F	Notes
1	113	38.2	15.1	0.13	76	20	6	23	0.27	high
2	117	29.8	9.0	0.19	72	26	4	23	0.21	
3	125	12.9	6.2	0.06	68	30	3	23	0.091	
4	134	20.1	6.9	0.16	65	33	11	23	0.143	
5	142	20.1	6.2	0.19	60	44	5	18	0.143	
6	150	18.0	6.6	0.15	56	49	6	20	0.128	
7	159	27.9	8.1	0.21	53	55	5	21	0.199	
8	180	11.4	3.9	0.16	47	60	5	22	0.087	
9	187	23.7	7.4	0.19	45	65	5	24	0.169	
10	200	11.0	3.8	0.17	42	70	4	24	0.078	
11	217	19.8	6.2	0.19	39	74	11	27	0.141	
12	280	20.1	6.4	0.18	30	85	8	29	0.143	
13	342	18.3	6.3	0.16	24.3	93	6	28	0.130	
14	550	10.2	2.8	0.22	15.1	99	3	25	0.072	

Test with artificial ice: No. 3/9.2

Type: Inclined wedge $\alpha = 51.6^{o}$ $\beta = 60^{o}$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.91$ g/cm^3

$e = 1.0$ cm $\mu = 0.19$ $C_1/C_2 = 0.242$ $C_5/C_2 = 1.686$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{140.64}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc. coef. C_F	Notes
1	96	24.0	9.4	0.13	79	2	5	26	0.171	
2	100	10.5	4.9	0.07	75	7	5	23	0.075	
3	104	19.4	6.1	0.18	72	12	5	20	0.138	
4	114	22.5	6.5	0.21	65	17	7	20	0.160	
5	133	23.5	8.2	0.16	57	24	11	27	0.167	
6	144	31.0	8.7	0.21	48	35	4	27	0.220	
7	163	17.7	5.2	0.20	42.5	39	6	24	0.126	
8	185	21.6	6.2	0.21	37.5	45	6	23	0.154	
9	225	17.1	4.9	0.21	30.7	51	3	19	0.121	
10	281	16.8	4.1	0.24	24.5	54	4	18	0.120	
11	407	9.5	2.7	0.21	17.0	58	3	18	0.068	

Test with artificial ice: No. 3/10.1

Type: Inclined wedge $\alpha = 51.6^{o}$ $\beta = 60^{o}$ $d = 16$ cm

Ice: $r_b = 1.92$ kg/cm^2 $r_c = 8.79$ kg/cm^2 $\rho = 0.83$ g/cm^3

$e = 1.0$ cm $\mu = 0.18$ $C_1/C_2 = 0.254$ $C_5/C_2 = 1.757$

$$C_F = \frac{F'}{r_c \cdot e \cdot d} = \frac{F'}{140.64}$$

1	2	3	4	5	6	7	8	9	10	
Encounter no.	System con. C	Horizontal force F' kilogram	Vertical force V' kilogram	Friction coef. μ	Velocity U_c cm/sec	Distance Σ x cm	Break off dist. x cm	W. of rupt. channel in cm	Reduc.coef. C_F	Notes
1	117	21.7	6.9	0.18	66	20	5	18	0.154	
2	121	26.2	10.5	0.12	65	25	6	18	0.186	
3	125	19.4	7.0	0.15	63	31	6	18	0.138	
4	129	14.5	5.4	0.14	61	37	9	18	0.103	
5	133	14.2	4.2	0.20	59	46	5	18	0.101	
6	140	18.3	5.8	0.18	55	51	6	23	0.130	
7	156	17.6	6.1	0.16	50	57	4	25	0.125	
8	165	21.0	5.9	0.21	47	61	3	28	0.150	
9	177	20.1	5.9	0.20	44	64	4	23	0.143	
10	196	15.8	5.4	0.17	40	68	3	22	0.112	
11	209	14.0	4.5	0.18	37.5	71	4	18	0.100	
12	225	18.7	6.3	0.17	34.6	75	5	16	0.133	?
13	287	19.4	5.6	0.20	28.0	80	4	16	0.138	?
14	538	13.6	3.9	0.20	14.5	84	4	20	0.097	

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